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# The ultra Nova model of cosmic Gamma Ray bursts

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**Abstract** : We point out that several stiff nuclear equation of states allow existence of Ultra Compact Neutron Stars (UCNS) whose surface gravitational redshifts ($z$) could be much higher than that of a canonical Neutron Star ($z \approx 0.19$). We show that even though the energy released in the form of $\nu - \bar{\nu}s$ during the formation of UCNSs rises modestly as $Q_\nu \sim z$, the energy of the resultant electromagnetic fireball $\nu + \bar{\nu} \rightarrow e^- + e^+$ rises dramatically as $Q_{FB} \sim z^{2.25}(1+z)^{4.5}$. Accordingly, we outline here the (new) class of model for the luminous ($Q_\gamma \geq 10^{52}$ erg) Gamma Ray Bursts associated with birth of Ultra Compact Objects. Even if we restrict ourselves to a value of $z \leq 0.615$, which is appropriate if the sound speed within the compact object is restricted to be $c_s \leq c/3$, where $c$ is the speed of light, this model may yield a maximum value of $Q_\gamma \approx 10^{53}$ erg.

**Keywords** : Gamma ray bursts, neutron stars, relativity and gravitation

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Following the observational revolution triggered by Beppo-Sax, it is now clear that a large number of Gamma Ray Bursts (GRBs) involve emission of $\gamma$-rays as large as $Q_\gamma \sim 10^{52} - 10^{54}$ erg under condition of isotropy. If the rapid fading of the optical afterglow for several bursts is interpreted due to beaming effects [1], the actual gamma ray energy involved could be lower approximately by 2 orders of magnitude. However, such rapid fading may also occur if the afterglow propagates in a very thick wind [2] or in a wind with a density gradient [3]. Also, in case of blazar type beamed emission, one may expect fairly high (several %) to very high degree of linear polarization (upto ~40%). But, polarization has so far been detected for only one instance (GRB 990510) and it has a small value of 1.7% [4]. Further, most of the afterglows do not show and unusually rapid fading and the decay of the light curve may be smoothly fitted by a single power on the time scale of months. Such GRBs are definitely expected to be more or less spherical events, and thus, for GRB 971214, we indeed have $Q_\gamma \approx 3 \times 10^{53}$ erg. In the following, we endeavour to explain the origin of GRBs with a value of $Q_\gamma \sim 10^{53}$ erg as a result of birth of likely ultra compact neutron stars (UCNS).

General Theory of Relativity (GTR) yields an absolute upper limit on the value of the surface gravitational redshift of a *static* relativistic spherical star [5] :

$$z = \left(1 - \frac{2GM(R)}{Rc^2}\right)^{-1/2} - 1 \leq z_c = 2. \qquad (1)$$

Here, $r$ is the invariant circumference radius, $c$ is the speed of light, and $M$ is the gravitational mass enclosed within $r = r$

$$M(r) = \int_0^r \rho dV = \int_0^r dM, \qquad (2)$$

where $\rho$ is the total mass-energy density, $dV = 4\pi r^2 dr$ is coordinate volume element, and the symbol $dM$ is self-explanatory. This result is obtained when the Equation of State (EOS) is allowed to have a causality violating sound speed $c_s = (dp/d\rho)^{1/2} > c$. When the EOS is constrained to obey causality, it follows from eq. (9.5.19), (p 261) of Ref [6], that one would have a tighter limit on $z_c = 1.22$. If one constrains the EOS further so as to have $c_s \leq c/\sqrt{3}$, it follows that one has an even tighter bound on $z_c = 0.615$ [7]. Note that although the *presumed* canonical NS has a value of $M \sim 1M_\odot$ (solar mass) and $R \sim 10$ km with $2M/R \sim 0.29$ ($G = c = 1$) and $z \sim 0.19$, many stiff *causality obeying* EOSs actually allow the existence of NSs with much higher value of $z$ [8]. The latter EOS gives a maximum value $M_m \approx 2.9M_\odot$ with a corresponding value of $R \approx 12$ km (violation of causality leads to still considerable higher value of $M_m$). However, in Figure 1, we show the plots of $M_m$, $R$ and $M_m/R$, for a certain fiducial density $\rho_0 = 2$ nuclear density, from the *updated version* of the previous work as kindly supplied to us by Kalogera and Baym [9]. If we restrict ourselves to $z_c = 0.615$ (the horizontal line in Figure 1, the saturated plateau region would yield $M_m \approx 2.2\ M_\odot$ and $R \approx 10.1$ Km The self-gravitational energy of a static relativistic star is given by Weinberg [5].

$$E_g = \int \rho dV \left\{ 1 - \frac{2M(r}{} \right. \qquad (3)$$

Then recalling the definition of $z$ from eq. (1), we may write

$$E_g = -\int z(r) dM \approx \alpha z M \approx -zM, \qquad (4)$$

where $\alpha \approx 1$ is a model dependent parameter (we have numerically verified that for $z < 1$, indeed $\alpha \approx 1$). The binding energy, *i.e.*, the energy liberated in the formation of the eventually cold stellar mass compact object, is given by virial theorem to be $E_B = (1/2)\,|E_g|$. Most of this binding energy is expected to be radiated in the form of $\nu - \bar{\nu}$ during the *final stages* of formation of the UCNS : $Q_\nu \approx E_B \approx \frac{zM}{2}$.

So, given the most restricted limit $z_c = 0.615$ the minimum value of $Q_\nu \approx 0.6 M_2 \approx 1.2 \times 10^{54} M_2$ erg where of $M = M_2 2 M_\odot$. This is in agreement with our similar previous crude estimate [10] The value of $Q_\nu$ measured near the compact object will be higher by a factor $(1+z) \cdot Q_\nu = z(1+z)M/2$. For the NS-formation case, the neutrinos diffuse out of the hot core in a time $t_\nu < 10$ s and we may expect a somewhat longer time scale for the diffusion of neutrinos from the nascent hot UCNS. However, here note that, the rather long value of $t_\nu < 10$ s occurs because of *coherent scattering* of neutrinos by the heavy (Fe) nuclei [6]. If the Fe-nucleons are already *partially dissociated* by an immediately preceding heating, the rise in the value of $t_\nu$ for the UCNS formation need not be much larger. And the locally measured duration of the burst would be $t'_\nu \sim (1+z)^{-1} t_\nu$. Therefore, the mean (local) $\nu - \bar{\nu}$ luminosity will be

$$L'_\nu = \frac{Q'_\nu}{t'_\nu} = \frac{z(1+z)^2 M}{2 t_\nu} \approx 2 \times 10^{53} z(1+z)^2 M_2 t_{10}^{-1} \text{ erg/s}, \quad (5)$$

where $t_\nu = t_{10} 10$ s. It may be noted that this value of $L'_\nu$ is well below the corresponding $\nu$-Eddington luminosity. The luminosity in each species will be $L'_i = (1/6) L'_\nu$. By assuming the radius of the neutrinosphere to be $R_\nu = R$, the value of effective local neutrino temperature $T$ (assumed to be same for all the flavors), is obtained from the condition

$$L'_\nu = \frac{21}{8} 4\pi R^2 \sigma T'^4, \qquad (6)$$

where $\sigma$ is the Stefan-Boltzman constant. Therefore, we have

$$T' = \left( \frac{2z(1+z)^2 Mc^2}{21 \pi \sigma R^2 t_\nu} \right)^{1/4}$$

$$\sim 13.3 MeV z^{0.25} (1+z)^{0.5} M_2^{0.25} R_6^{0.5} t_{10}^{0.25}, \quad (7)$$

where $R = R_6 10^6$. For a Fermi-Dirac distribution, under the crude assumption of zero $\nu$-chemical potential, the mean (local) energy of the neutrinos is $E'_\nu = 3.15T \sim 48$ MeV (for $z = 0.6$). The various neutrinos will collide with their respective antiparticles to produce electromagnetic pairs by the $\nu + \bar{\nu} \to e^+ + e^-$ process. The rate of energy generation by pair production per unit volume per unit time, at a distance $r$ from the center of the star, is given by Goodman *et al* [11] and Dar *et al* [12] :

$$q_\pm(r) = \sum \frac{K_{\nu i} G_F^2 E'_\nu L_i'^2(r)}{12 \pi^2 c R_\nu^4} \varphi(r)$$

Here, $L'_i(r) \sim r^{-2}$ is the $\nu$-flux density of a given species above the $\nu$-sphere, $G_F^2 = 5.29 \times 10^{-44}$ cm$^2$ MeV$^{-2}$ is the universal Fermi weak coupling constant squared, $K_{\nu i} = 2.34$ for electron neutrinos and has a value of 0.503 for muon and tau neutrinos. Here, the geometrical factor $\varphi(r)$ is

$$\varphi(r) = (1-x)^4 (x^2 + 4x + 5); \; x = [1 - (R_\nu / r)^2]^{1/2} \quad (9)$$

Now, considering all the 3 flavors, a simple numerical integration yields the local value of pair luminosity produced above the neutrinosphere :

$$L'_\pm = \int_R^\infty q_\pm 4\pi r^2 dr \approx \sum \frac{K_{\nu i} G_F^2 E'_i L_i'^2}{27 \pi c R_\nu}$$

$$\approx 2 \times 10^{51} z^{2.25} (1+z)^{4.5} M_2^{2.25} t_{10}^{-2.25} R_6^{-2} \text{ erg/s} \quad (10)$$

This estimate is obtained by assuming rectilinear propagation of neutrinos near the UCNS. Actually, in the strong gravitational field near the UCNS surface the *neutrino orbits will be curved* with significant higher effective interaction cross section. Since, most of the interactions take place near the $\nu$-sphere, for a modest range of $z$, we may tentatively try to incorporate this nonlinear effect by inserting a $(1+z)$ factor in the above expression On the other hand, the value of this electromagnetic luminosity measured by a distant observer will be smaller by a factor of $(1+z)^2$, so that eventually, $L_\pm - L'_\pm$ of eq (11). And the total energy of the electromagnetic Fire Ball (FB) at $\infty$ is

$$Q_{FB} = t_\nu L_\pm \approx 2 \times 10^{52} z^{2.25} (1+z)^{4.5} M_2^{2.25} t_{10}^{-1.25} R_6^{-2} \quad (11)$$

If we restrict $z_c \le 0.615$, the optimal values for a UCNS are $M_2 = 1.1$ and $R_6 \approx 1.1$. Then, for $t = 8s$, we obtain a highest value of $Q_{FB} \approx 10^{53}$ erg. The *efficiency* for conversion of $Q_\nu$ into $Q_{FB}$ in this case is $\varepsilon_\pm = Q_{FB} / Q_\nu \approx 5\%$. If we relax the condition, $z \le 0.615$ and only apply the constraint that $c_s < 1$, it is possible to explain a much higher value of $Q_{FB} \sim few\ 10^{53}$ erg because these Kalogera-Baym EOS yields higher values of $M_m$ and $z_c$ for lower values of $\rho_0$ and $\rho_c$.

We need the degree of baryonic pollution $\eta = Q_{FB} / \Delta M > 10^2$, where $\Delta M$ is the mass of entrained baryons In general, all models involving collision and full/partial disruption of compact object(s) will spew out thick and massive debris (few $M_\odot > M_* > 0.1 M_\odot$). Part of this debris is likely to settle into a torus and an uncertain small fraction ($\Delta M$) may hang around the system and get accreted on a long time scale or may even by unbounded. It is practically, impossible to simulate the latter fraction *dynamically* even in a Newtonian theory. On the other hand, *quasi-spherical implosion models* are free from the presence of such unaccountable and intractable thick collisional debris.

In a normal Supernova (SN) event (assumed to be basically spherical implosion), the ejection of baryonic mass $\sim 0.1 M_\odot$ occurs probably because of shock mediated hydrodynamic process. Since, by definition, the system is *gravitationally bound*, any normal hydrodynamic attempt of mass ejection can not be much successful in a spherical model. But the shock generates additional entropy and heat in its vicinity and might be able to effect the mass ejection. Yet, the shock is constantly depleted of energy and gets stalled because of $\nu$-losses and disintegration of heavy nuclei [13]. Probably, the shock might be rejuvenated by the "shock reheating mechanism". The energy transfer between neutrinos and matter behind the shock is mediated primarily by the charged current reactions $\nu_e + n \rightarrow p + e^-$ and $\bar{\nu}_e + p \rightarrow n + e^+$. When these reactions proceed to the right, the matter heats up, and conversely, the matter cools. To have a successful and sufficient net heating is a critical phenomenon, and present day (realistic) SN codes are unable to find the shock mediated mass-ejection (explosion) even in a relatively *weak* nascent-NS gravitational field [14]. And the basic reason that a *critical phenomenon* like shock heated mass ejection might be successful for the SN case is that as one moves from a relativistic potential well (high $z$) to a Newtonian well ($z \le 0.2$), the local temperature due to $\nu$-heating may decrease slowly $T' \sim z^{0.25}$ and the $\nu$-matter interaction cross-section $\sigma_{\nu,m} \sim T'^2 \sim z^{0.5}$, while, the depth of the potential well drops rapidly $\propto z$. Note that an UCNS with a modest value of $z \sim 0.615$ has a potential well which is $\sim 300\%$ deeper than the one associated with a canonical NS, $z \sim 0.19$.

There is a genuine possibility, that all models of cosmological GRBs, irrespective of whether they explicitly invoke the $\nu + \bar{\nu} \rightarrow e^+ + e^-$ process or not, should involve strong direct electromagnetic or $\nu$ – *heated* mass loss. For instance, even if an unusual pulsar is assumed to emit $\sim 10^{52}$ erg/s, *the superstrong return current impinging back on the pulsar* may drive a catastrophic wind, a possibility not considered so far by any author. On the other hand, for the case of neutrino mediated GRBs, for the thin outermost layers of the object (UCNS or an hot accretion torus) emitting the neutrinos, well above the $\nu$-sphere, the $\nu$-flux $S_\nu$ may induce a super-Eddington photon flux $S_{ph}$ [15]. Note that, although, for a torus with uncertain dynamically changing geometry, it is practically impossible to make any semi-analytical or numerical estimate of such a process, in general this effect is expected to be much more pronounced because its gravitational *self-binding* ($z$) is much weaker than that for a spherical UCNS surface. And even if a steady state model calculation yields a high value of $\eta$, the eventual value of $\eta$ might be very low if the jet is intercepted by this debris. Most of the GRB models which envisage a jet to emanate from the polar region of a compact object accreting matter from a transient disk suffer from yet another severe problem. Since we know that *the accreted matter eventually lands up near the polar region*, atleast at first sight, the *jet should be most baryon polluted* in such cases. In fact, (by ignoring all such unmanagable real life uncertainties and difficulties), detailed Newtonian and crude post Newtonian calculations for the NS-NS collision case have been presented by several authors [16]; and the conclusion is that, it is difficult to understand a value of $\eta$ higher than few.

The estimate of $\Delta M$ may be made with much larger confidence only for a spherical model, where by definition, *the entire matter, in general, is moving inwardly*. Probably, the most detailed work on this problem of $\bar{\nu}$-driven mass ejection from a hot nascent NS is due to Duncan *et al* [15], and the *Table 5* of it shows that for $R \approx 10^6$ cm, $M = 2M_\odot$, we have $\Delta M \approx 10^{-4} M_\odot$, if $T' = 20$ MeV. On the other hand, for $T' = 30$ MeV, one has, $\Delta M \approx 7 \times 10^{-4} M_\odot$.

The above mentioned estimates were made in the framework of Newtonian gravity, and a GTR calculation, if possible, would certainly yield, lower values of $\Delta M$. However, a proper GTR treatment of the problem would be an extreme difficult task, and, at present, we are aware of one post-Newtonian calculation to this effect. Note that, in a Newtonian calculation, the $\nu$-driven mass loss rate should change only modestly with the value of $2\,GM/R$. However, one may note from Figure 3 of Cardall and Fuller [17] that, in a post-Newtonian calulation, the value of $\dot{M}$ *decreases dramatically* as one increases the value of $2\,GM/R$. For instnce, while $\dot{M} \sim 10^{-5} M_\odot$/s for $2\,GM/R = 0.2$, one has $\dot{M} \sim 10^{-7}\,M_\odot$/s for $2\,GM/R = 0.6$. And thus, it may indeed be possible to have $10^3 > \eta > 10^2$ in the high $z$ case. Now the occurrence of luminous and long GRBs might be understood either by considering internal dissipation of the FB or if the FB interacts with a very dense ambient medium [18]. It is possible that such UCNS configurations are *not stable*. If so, with little perturbation, such an UCNS, having yielded the GRB, may proceed for further collapse and releases additional energy. Similarly, this limiting stage $z = z_c$ might be preceded by one or more metastable stages of less compact NSs. If it is indeed so, for certain rare cases, the GRB event might be preceded by a Supernova event occurring in a low $z$ region. If the UCNS is *ultramagnetized* too and spins fast, it is plausible that the *electromagnetic* FB will be beamed, and then, the present model may explain the origin of beamed GRBS with energy up to $\sim$ *few* $10^{53}/4\pi$ erg/sr. But as mentioned earlier, in highly anisotropic cases, it is difficult to make any realistic estimate of the baryon pollution unlike the present case.

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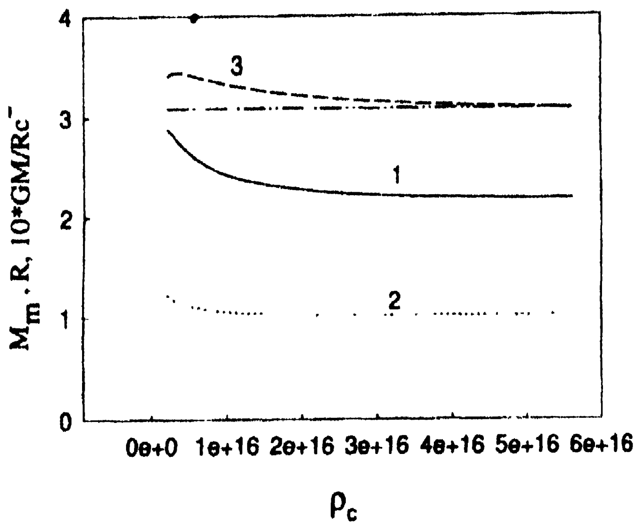


**Figure 1.** Plot of (1) $M_m$ in units of $M_\odot$, (2) the corresponding radius $R$ in units of 10 Km, (3) the corresponding value of $10 \times GM/Rc^2$ against central density, $\rho_c$ (gm/cm$^3$) The cutoff line corresponds to $GM/Rc^2 \approx 0.31$ ($z_c \approx 0.615$)

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